TdN - Sessione 2

Titolo nota 18/10/2017

$S = \{a_1, \dots, a_{2017}\}$, $a_i \in \mathbb{Z}$ $\forall i$

$\exists S$:

1) $\forall T \subseteq S$, $AM(T) \in \mathbb{Z}$?

$a_1 = a_2 = \dots = a_{2017}$

2) a_i distinti $\qquad \{2, 4, 10\} \longrightarrow \frac{16}{3}$

$T = \{b_1, \dots, b_k\} \rightsquigarrow AM(T) = \boxed{\dfrac{b_1 + \dots + b_k}{k}}$

$a_i = 2017! \cdot i$

3) $(a_i, a_j) = 1 \qquad MCD(a_i, a_j) \equiv (a_i, a_j)$

$b_1 + b_2 + \dots + b_k \equiv 0 \pmod k$

$a \equiv b \pmod c \iff a = ck + b$

$a_i \equiv 1 \ (k) \qquad k = 1, \dots, 2017$
$\qquad i = 1, \dots, 2017$

$a_i - 1 \equiv 0 \ (k) \implies a_i - 1 = 2017! \cdot i$

$a_i = 1 + 2017! \cdot i$

$i > j$

$(a_i, a_j) = (1 + 2017! \cdot i, \ 1 + 2017! \cdot j)$

$= (2017!(i-j), \ 1 + 2017! \, j) = 1$

$0 < i - j < 2017$

$p \mid 2017!(i-j)$

4) $\forall T \subseteq S \qquad GM(T) \in \mathbb{Z}$

$$T = \{b_1, \dots, b_k\} \rightsquigarrow GM(T) = \sqrt[k]{b_1 \cdot \dots \cdot b_k}$$

$$a_i = \left(1 + 2017! \, i\right)^{2017!}$$

$$b_1 + \dots + b_k \equiv 1 + 1 + \dots + 1 \equiv k \equiv 0 \ (k)$$

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ARITMETICA MODULARE

$a \equiv b \pmod{c} \qquad a = kc + b$

$$\left.\begin{aligned} a_1 &\equiv b_1 \pmod{c} \\ a_2 &\equiv b_2 \pmod{c} \end{aligned}\right. \longrightarrow \begin{cases} a_1 a_2 \equiv b_1 b_2 \ (c) \\ a_1 + a_2 \equiv b_1 + b_2 \ (c) \end{cases}$$

3 $\quad 15 \equiv 3 \ (4) \rightsquigarrow 5 \equiv 1 \ (4)$

2 $\quad 6 \equiv 2 \ (4) \ \not\rightsquigarrow \ 3 \equiv 1 \ (4)$

$$\frac{1}{3} \cdot 15 = 3x \cdot \frac{1}{3} \longrightarrow 5 = x$$

$$1/3 \equiv \ ? \ (4)$$

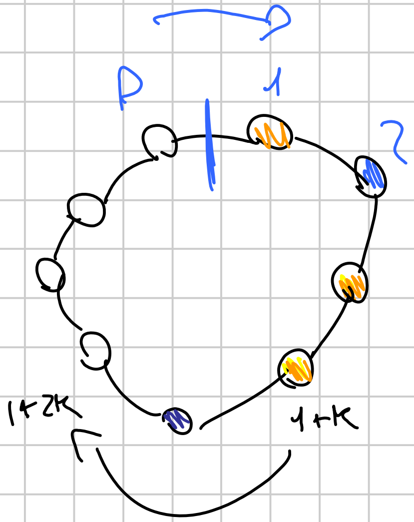
$$1/3 \equiv 3 \ (4) \qquad \{0, 1, 2, 3\}$$

$$(a, c) = 1 \implies \exists!\, b : \quad ab \equiv 1 \ (c)$$

Teorema di Fermat

$\forall p$ primo $\quad \forall a$ intero $\qquad \boxed{a^p \equiv a \pmod{p}}$

$(a, p) = 1 \qquad a^{p-1} \equiv 1 \pmod{p}$



$(a+1)^p \equiv a^p+1 \equiv a+1$

$p \mid \binom{p}{k} \qquad k \neq 0, p$

p perle

a colori

$a^p - \boxed{a} \equiv 0 \ (p)$

$(k,p)=1 \quad \forall\ 0<k<p$

$1 \longrightarrow 1+k \longrightarrow 1+2k \longrightarrow \dots\ 1+dk$

Ex $\quad p \neq 2,3 \quad p$ primo $\qquad 2^{p-2}+3^{p-2}+6^{p-2} \equiv 1 \ (p)$

$$2^{p-2}\cdot 2\cdot\frac{1}{2} + 3^{p-2}\cdot 3\,\frac{1}{3} + 6^{p-2}\cdot 6\,\frac{1}{6} \equiv 1 \ (p)$$

$$2^{p-2}\cdot 2\cdot\frac{1}{2} = 2^{p-1}\cdot\frac{1}{2} \equiv \frac{1}{2}$$

$$3^{p-2}\cdot 3\,\frac{1}{3} = 3^{p-1}\cdot\frac{1}{3} \equiv \frac{1}{3}$$

$$6^{p-2}\cdot 6\,\frac{1}{6} = \dots \equiv \frac{1}{6}$$

$$\boxed{p^a+p^b=n^3}$$

$a,b,p>0 \quad p$ primo

$a \geq b$

$a-b>0$

$p^b(p^{a-b}+1)=n^3$

$(p^b, p^{a-b}+1)=1$

$b=3k$

$p^{a-b}+1=m^3$

$p^{a-b}=m^3-1=(m-1)(m^2+m+1)$

1, 3

coprimi $\quad \boxed{m-1=1} \quad m=2 \quad p=7$

$3 \mid p^{a-b} \quad \cancel{m-1=3} \Rightarrow \cancel{p=3}$

$$\boxed{2p^a=n^3}$$

$p=2$

$$\boxed{\begin{aligned} a &= 2+3k \\ p &= 2 \\ n &= 2^{k+1}\end{aligned}}$$

$$(m-1,\ m^2+m+1) =$$
$$= (m-1,\ 1+1+1)$$

$$m^2 = (m-1+1)^2 = \boxed{(m-1)[\dots]} + 1$$
$$m = (m-1)+1 = \boxed{(m-1)} + 1$$
$$1 = 1 = 1$$

$$m^k = [(m-a)+a]^k =$$
$$= (m-a)[\dots] + a^k$$

$$(m-1,\ m^2+m+1) = (m-1,\ (m-1)[\ \ ]+3)$$
$$= (m-1,\ 3)$$

$x^2 \equiv ?\ (3)$

$x^2 \equiv ?\ (5)$

$$\begin{array}{ccc} x & & x^2 \\ 0 & \longrightarrow & 0 \\ 1 & \longrightarrow & 1 \\ 2 & \longrightarrow & 1 \end{array}$$

$$\begin{array}{ccc} x & & x^2 \\ 0 & \longrightarrow & 0 \\ 1 & \longrightarrow & 1 \\ 2 & \longrightarrow & -1 \\ 3 & \longrightarrow & -1 \\ 4 & \longrightarrow & 1 \end{array}$$

(p)

$$\begin{array}{ccccc|ccc} 1 & 2 & 3 & & \dots & p-3 & p-2 & p-1 \\ \downarrow & \downarrow & & & & & \downarrow & \downarrow \\ 1 & 4 & & & & & 4 & 1 \end{array}$$

x^2

$$\begin{array}{ccc} x & & x^3 \\ 0 & \longrightarrow & 0 \\ 1 & \longrightarrow & 1 \\ 2 & \longrightarrow & 1 \\ 3 & \longrightarrow & -1 \\ 4 & \longrightarrow & 1 \\ 5 & \longrightarrow & -1 \\ 6 & \longrightarrow & -1 \end{array}$$

$p = 7$ $\qquad$ $p-1 = 6$ $\qquad$ x^3

$\boxed{x^3}$ $\quad$ $\boxed{x^2}$ $\quad$ 3, 5, 7

$3 \mid p-1$

$x^5 \quad (11)$

$5 \mid 11-1$

$$x^5 \equiv x^3 \cdot x^2 \quad (3)$$
$$\equiv x \cdot x^2$$
$$\equiv x^3 \equiv x$$

Ex Quali tra questi sono somma di due quadrati

2004, 2005, 2006

$$2006 = x^2 + y^2$$

$$2 \equiv \underset{0/1}{x^2} + \underset{0/1}{y^2} \quad (4)$$

$$\boxed{1 + 1} \leftarrow$$

$$6 \equiv \underset{0/1/4}{x^2} + \underset{0/1/4}{y^2} \quad (8)$$

$$\boxed{4 \quad\quad 4}$$

mod 4

$0 \to 0$
$1 \to 1$
$2 \to 0$
$3 \to 1$

mod 8

$0 \to 0$
$1 \to 1$
$2 \to 4$
$3 \to 1$
$4 \to 0$
$5 \to 1$
$6 \to 4$
$7 \to 1$

0, 1, 4

$$2004 \equiv 0 \equiv \underset{0/1}{x^2} + \underset{0/1}{y^2} \quad (3)$$

$3 \mid 2004 \longrightarrow 3 \mid x \quad 3 \mid y$

$x = 3a \quad y = 3b$

$$2004 = 9(a^2 + b^2)$$

$$2005 = 5 \cdot 401 = (1+4)(1+400) = (1+40)^2 + (20-2)^2$$

$41^2 + 18^2$

a, b, c, d

$$\boxed{(a^2+b^2)(c^2+d^2) = (ac+bd)^2 + (ad-bc)^2}$$

$(ad+bc)^2 + (ac-bd)^2$

$$a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2 = a^2c^2 + b^2d^2 + \cancel{2abcd} + a^2d^2 + b^2c^2 - \cancel{2abcd}$$

$$\|\alpha \cdot \beta\| = \|\alpha\| \cdot \|\beta\|$$

$\alpha = a + ib$

$\beta = d + ic$