TdN - Esercizi

Titolo nota 18/10/2017

Ex 1 (p,q) : $p^3 - q^5 = (p+q)^2$

(mod 3) $p^3 \equiv p \ (3)$ $q^5 \equiv q^3 \cdot q^2 \equiv q \cdot q^2 \equiv q \ (3)$

$p - q \equiv (p+q)^2 \pmod 3$ $\begin{cases} \equiv 0 \ (3) \\ \equiv 1 \ (3) \end{cases}$

- $p \equiv q \ (3) \implies (p+q)^2 \equiv (2p)^2 = 4p^2 \ (3)$
  $\equiv p^2 \equiv 0 \ (3)$
  $\implies p = q = 3$ $3^3 - 3^5 < 0$
- $p \equiv q+1 \ (3)$ $(p+q)^2 \equiv 1 \ (3)$

$$\begin{array}{cc} \boxed{1 \quad 0} & \to 1 \\ 2 \quad 1 & \times \\ \boxed{0 \quad 2} & \to 1 \end{array}$$

$p = 3 \to 3^3 - q^5 > 0$ $27 - 32 < 0$ $\times$

$q = 3 \to \boxed{p^3 - 3^5 = (p+3)^2}$ $p < 10$ $p = 5$ $\times$ $\boxed{p = 7, q = 3}$

$1000 - 243 \not> 169$

$343 - 243 = (7+3)^2$

Ex 2 $(a_1, \dots, a_n)$ $(a_1, a_1+1, a_1+2, \dots, a_1+n-1)$

$\boxed{1}$ $(a_i)_{i=1,\dots,n}$ $(a_i, b_i) = 1$ $\forall i$

$(b_i)_{i=1,\dots,n}$

$((n+1)! + k-1, k) = 1$

$(1, 2, 3, 4, \dots, n)$

$(\cdot, \cdot, \cdot, \quad \cdot)$ $((n+1)!, (n+1)!+1, (n+1)!+2, \dots, \dots)$

$\boxed{2}$ $(a_i, b_i) > 1$ $\forall i$

$(a_i, b_i) = a_i = i+1$

$(2, 3, \dots, n+1)$

$((n+1)!+2, (n+1)!+3, \dots, (n+1)!+n+1)$

Ex 3 (x, y, z, w) $\quad 2^x + 3^y + 5^z = 7^w$ $\quad a \in \mathbb{N}$

(mod 2) $\quad 2^x + 1 + 1 \equiv 1$

$$2^x \equiv 1 \implies \underline{x = 0}$$

$$1 + 3^y + 5^z = 7^w$$

(mod 3) $\quad 1 + 3^y + (-1)^z \equiv 1$

$$3^y \equiv -(-1)^z \begin{cases} 1 \\ -1 \end{cases} \Rightarrow \underline{y = 0}$$

$$2 + 5^z = 7^w \qquad \begin{matrix} x & y & z & w \\ (0, & 0, & 1, & 1) \end{matrix}$$

$z > 1 \Rightarrow z \geq 2$

(mod 25) $\quad 2 \equiv 7^w$

$7^1 \equiv 7$

$7^2 = 49 \equiv -1$

$7^3 \equiv -7$

$7^4 \equiv 1$

$\Rightarrow$ NON esistono soluzioni con $z \geq 2$

Ex 4 $\quad x_1 = 4 \qquad x_{n+1} = x_1 \cdot x_2 \cdot \ldots \cdot x_n + 5$

$1 \leq a < b \qquad x_a \cdot x_b$ quadrato perfetto?

$\boxed{x_1 = 4} \quad \boxed{x_2 = 9} \quad \underline{x_3 = 41} \qquad x_1 \cdot x_2 = 4 \cdot 9 = 36 = 6^2$

$(x_a, x_b) = ?$ $\qquad b > a \Rightarrow (x_b, x_a) =$

$$= (x_1 \cdot x_2 \cdot \ldots \cdot x_{b-1} + 5, x_a)$$

$$= (5, x_a) \begin{cases} 1 \\ 5 \end{cases}$$

- $5 \mid x_a = 5 + \overbrace{x_1 \cdot \ldots \cdot x_{a-1}} \rightsquigarrow 5 \mid x_k \quad k < a$

  $5 \mid x_a \Rightarrow \ldots \Rightarrow \ldots \Rightarrow 5 \mid x_1 = 4$ $\quad$ ✗

- $(x_a, x_b) = 1$, $\; x_a \cdot x_b = n^2 \Rightarrow x_a = k^2$, $\; x_b = h^2$

$$\begin{array}{cccccc} 4 & 9 & 9\cdot 4+5 & 9\cdot(\ )+5 & & 9(\ )+5 \\ x_1 & x_2 & x_3 & x_4 & \ldots & x_k \\ 1 & 0 & 2 & 2 & \ldots & 2 \end{array} \quad (3)$$

$$x^2 \equiv 0/1 \ (3)$$

Ex 5

$$n^5 + n^4 = 7^m - 1 \qquad (n, m)$$

$$\underbrace{n^5 + n^4 + 1}_{\downarrow} = 7^m \qquad n^3(n^2+n) + 1 = 7^m$$

$$n^3(n^2+n+1) + \underbrace{1 - n^3}_{} = 7^m$$

$$(n^3 - n + 1)(n^2 + n + 1) = 7^m$$

$$7^a \qquad 7^b$$

$n^3 - n + 1 = 1 \longrightarrow n = 1$ ✗

$n^2 + n + 1 = 1 \longrightarrow \boxed{n = 0}$ $\boxed{m = 0}$

$n^2 + n + 1 = 7 \longrightarrow \boxed{n = 2}$ $\boxed{m = 2}$

$n^3 - n + 1 = 7 \longrightarrow n = 2$ "

$$(n^3 - n + 1, n^2 + n + 1)$$

$$= (n^3 - n + 1 - n(n^2+n+1), n^2+n+1)$$

$$= (-n^2 - 2n + 1, n^2 + n + 1) =$$

$$= (-n + 2, n^2 + n + 1) = (n-2, n^2+n+1)$$

$$= (n-2, 7) < \begin{matrix} 1 \\ 7 \end{matrix}$$

$$n^2 + n + 1 = (n-2)q(n) + r$$

$$7 = r$$

Ex 6 (x, y, z) $\quad 2^x + 3^y = z^2$

$x = 0 \longrightarrow 3^y + 1 = z^2 \longrightarrow 3^y = (z-1)(z+1)$

$$(z-1, z+1) = (2, z+1) < \begin{matrix} 2 \\ 1 \end{matrix}$$

$z - 1 = 1 \rightarrow z = 2$

$z + 1 = 1 \rightarrow z = 0$

$$\boxed{(0, 1, 2)}$$

$y = 0 \longrightarrow 2^x + 1 = z^2 \longrightarrow 2^x = (z-1)(z+1)$

$$(z-1, z+1) = (2, z+1) < \begin{matrix} \boxed{2} \\ \boxed{1} \end{matrix}$$

$z - 1 = 2 \rightarrow z = 3$

$z + 1 = 2 \rightarrow z = 1$

$$\boxed{3, 0, 3}$$

$x, y > 0 \qquad 2^x + 3^y = z^2 \qquad \pmod 2 \longrightarrow 1 \equiv z^2$

$\pmod 3 \qquad 2^x \equiv z^2 \equiv \begin{cases} 0 & \text{NO} \\ \boxed{1} \end{cases} \qquad 2^x \equiv 1 \pmod 3$

$\Longrightarrow \quad 2 \mid x \longrightarrow \quad x = 2a$

$a > 0$

$2^1 = 2$
$2^2 = 1$
$2^3 = 2$
$2^4 = 1$
$\vdots$

$$3^y = z^2 - 2^{2a}$$
$$= (z - 2^a)(z + 2^a)$$

$$(z - 2^a, z + 2^a) = (z - 2^a, \underbrace{2 \cdot 2^a}) = 1$$

$z + 2^a = 1$ NO $\longrightarrow \begin{cases} z - 2^a = 1 \\ z + 2^a = 3^y \end{cases}$

$$2 \cdot 2^a = 3^y - 1$$

$$\boxed{2^{a+1} = 3^y - 1} \qquad a > 0$$

[Cultura Generale:
MIHAILESCU $\qquad x^a - y^b = 1$
$9 - 8$]

$\pmod 4 \qquad 0 \equiv 3^y - 1 \longrightarrow 3^y \equiv 1 \ (4)$

$3^1 = 3$
$3^2 = 1$

$\longrightarrow y = 2b$

$$2^{a+1} = 3^{2b} - 1 = (3^b - 1)(3^b + 1)$$

$$(3^b - 1, 3^b + 1) = (3^b - 1, 2) \begin{cases} 1 \\ 2 \end{cases}$$

$3^b - 1 = 1 \longrightarrow 3^b = 2 \ \times$

$3^b + 1 = 1 \longrightarrow 3^b = 0 \ \times$

$3^b - 1 = 2 \longrightarrow b = 1$

$x = 4 \quad y = 2 \quad z = 5$

$$\boxed{4, 2, 5}$$